Today you will:

- Describe transformations of quadratic functions
- Write transformations of quadratic functions
- Practice using English to describe math processes and equations

Core vocabulary:

- Quadratic function
- Parabola
- Vertex of a parabola
- Vertex form of a quadratic function

Describing transformations of quadratic functions … do we have it down?

$g(x) = (x - 3)^2$

Only has 1 transformation…

Subtracting inside means…
…translate right 3

$g(x) = (x - 2)^2 + 2$

Has 2 transformations…

Subtracting inside means…
…translate right 2

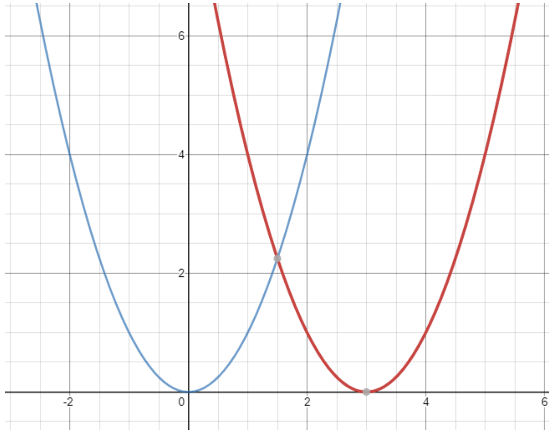
Adding outside means…
…up 2

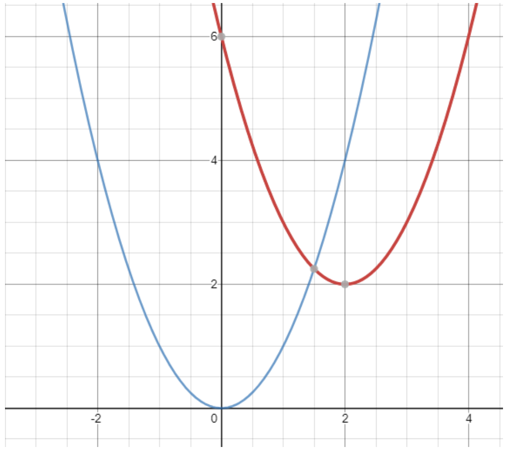
$g(x) = (x + 5)^2 - 2$

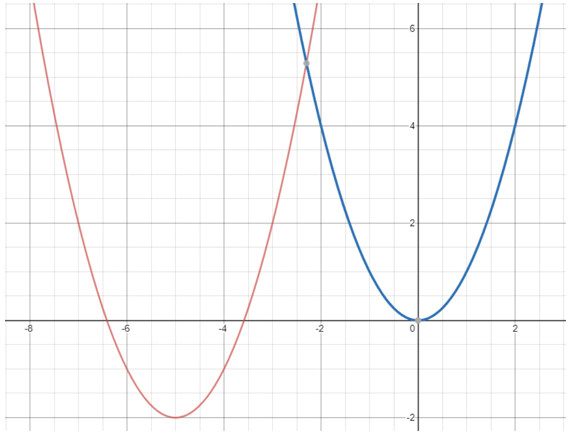
Has 2 transformations…

Adding inside means…
…translate left 5

Subtracting outside means…
…down 2





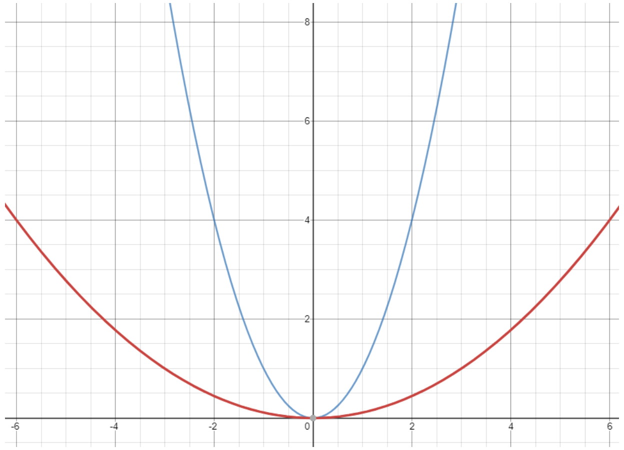


Describing transformations of quadratic functions … do we have it down?

$g(x) = (\frac{1}{3}x)^2$

Only has 1 transformation…

Multiplying fraction inside means…
…horizontal stretch factor 3

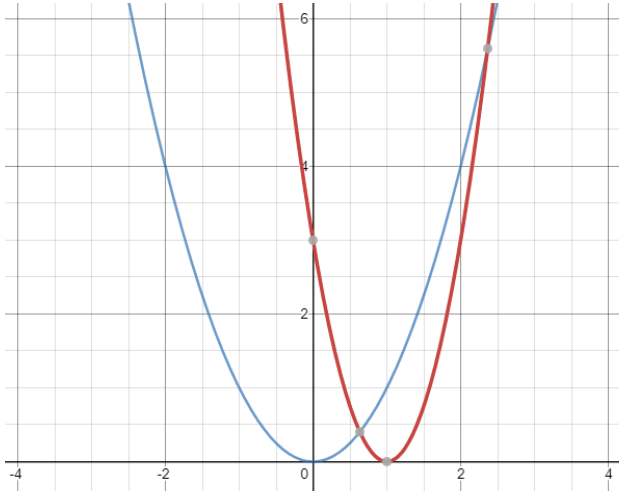


$g(x) = 3(x-1)^2$

Has 2 transformations…

Multiplying outside >1 means…
…vertical stretch factor 3

Subtracting inside means…
…translate right 1



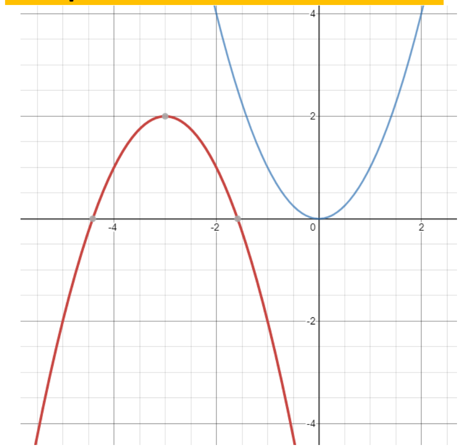
$g(x) = -(x+3)^2 + 2$

Has 3 transformations…

Negative outside means…
…reflect around x-axis

Adding inside means…
…translate left 3

Adding outside means…
…up 2



Parabola

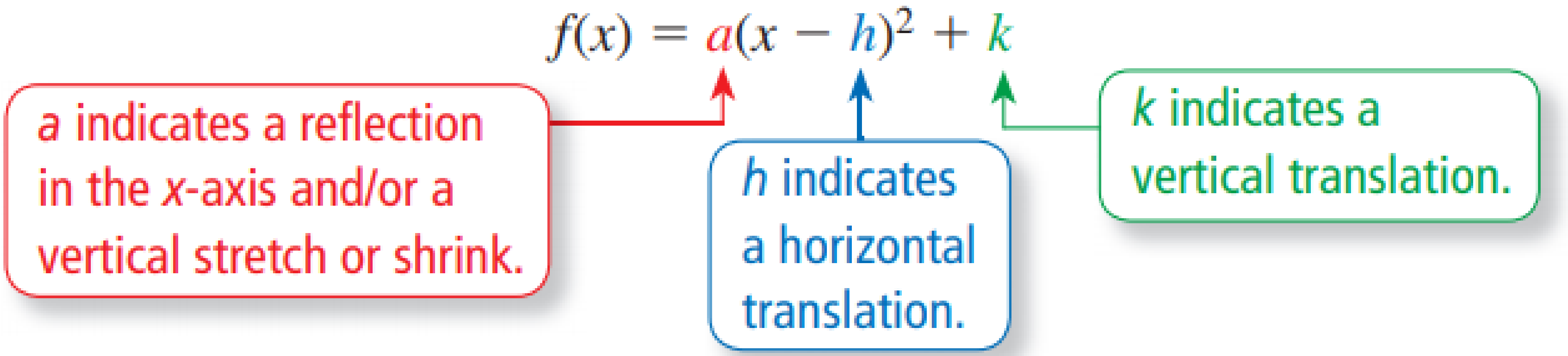
- Another name for the graph (shape) of a quadratic function

Vertex of a parabola

- Where the parabola bends / changes direction
- For a parabola that opens up, it is the lowest point (minimum y value)
- For a parabola that opens down, it is the highest point (maximum y value)

Vertex form of a quadratic function

$\boldsymbol{f(x) = a(x - h)^2 + k}$, where $a \neq 0$ and the vertex is (h, k).



Example 3

Let the graph of g be a vertical stretch by a factor of 2 and a reflection in the x-axis, followed by a translation 3 units down of the graph of $f(x) = x^2$. Write a rule for g and identify the vertex.

SOLUTION

Method 1 Identify how the transformations affect the constants in vertex form.

reflection in x-axis
vertical stretch by 2 } $a = -2$

translation 3 units down } $k = -3$

Write the transformed function.

$g(x) = a(x - h)^2 + k$ Vertex form of a quadratic function

$= -2(x - 0)^2 + (-3)$ Substitute -2 for a, 0 for h, and -3 for k.

$= -2x^2 - 3$ Simplify.

The transformed function is $g(x) = -2x^2 - 3$. The vertex is $(0, -3)$.

Method 2 Begin with the parent function and apply the transformations one at a time in the stated order.

First write a function h that represents the reflection and vertical stretch of f.

$h(x) = -2 \bullet f(x)$ — Multiply the output by -2.

$= -2x^2$ — Substitute x^2 for $f(x)$.

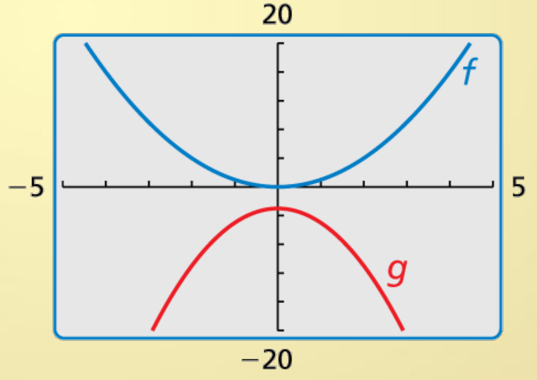
Then write a function g that represents the translation of h.

$g(x) = h(x) - 3$ — Subtract 3 from the output.

$= -2x^2 - 3$ — Substitute $-2x^2$ for $h(x)$.

The transformed function is $g(x) = -2x^2 - 3$. The vertex is $(0, -3)$.

Check



Example 4

Let the graph of g be a translation 3 units right and 2 units up, followed by a reflection in the y-axis of the graph of $f(x) = x^2 - 5x$. Write a rule for g.

SOLUTION

Step 1 First write a function h that represents the translation of f.

$$h(x) = f(x - 3) + 2$$ Subtract 3 from the input. Add 2 to the output.

$$= (x - 3)^2 - 5(x - 3) + 2$$ Replace x with $x - 3$ in $f(x)$.

$$= x^2 - 11x + 26$$ Simplify.

Step 2 Then write a function g that represents the reflection of h.

$$g(x) = h(-x)$$ Multiply the input by -1.

$$= (-x)^2 - 11(-x) + 26$$ Replace x with $-x$ in $h(x)$.

$$= x^2 + 11x + 26$$ Simplify.

REMEMBER

To multiply two binomials, use the FOIL Method.

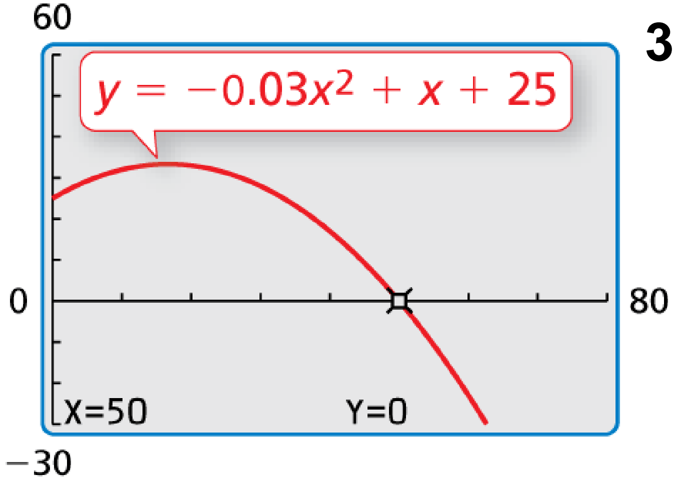
First Inner

$$(x + 1)(x + 2) = x^2 + 2x + x + 2$$

Outer Last

Example 5

The height h (in feet) of water spraying from a fire hose can be modeled by $h(x) = -0.03x^2 + x + 25$, where x is the horizontal distance (in feet) from the fire truck. The crew raises the ladder so that the water hits the ground 10 feet farther from the fire truck. Write a function that models the new path of the water.

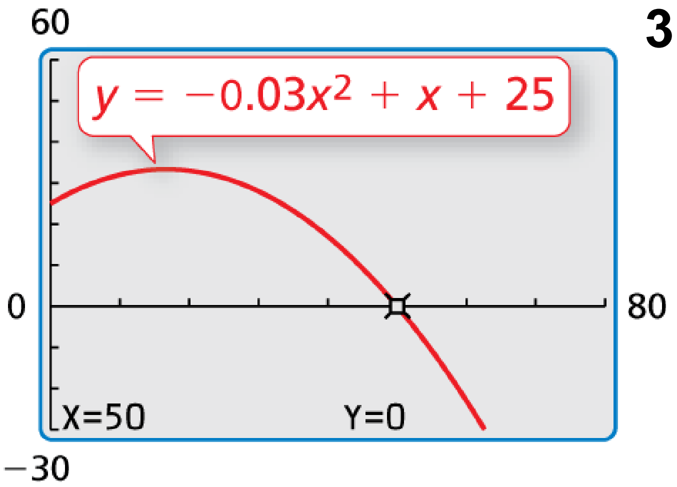


SOLUTION

1. **Understand the Problem** You are given a function that represents the path of water spraying from a fire hose. You are asked to write a function that represents the path of the water after the crew raises the ladder.

2. **Make a Plan** Analyze the graph of the function to determine the translation of the ladder that causes water to travel 10 feet farther. Then write the function.

3. **Solve the Problem** Use a graphing calculator to graph the original function. Because $h(50) = 0$, the water originally hits the ground 50 feet from the fire truck. The range of the function in this context does not include negative values. However, by observing that $h(60) = -23$, you can determine that a translation 23 units (feet) up causes the water to travel 10 feet farther from the fire truck.



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$$g(x) = h(x) + 23 \qquad \text{Add 23 to the output.}$$

$$= -0.03x^2 + x + 48 \qquad \text{Substitute for } h(x) \text{ and simplify.}$$

The new path of the water can be modeled by $g(x) = -0.03x^2 + x + 48$.

4. Look Back To check that your solution is correct, verify that $g(60) = 0$.

$$g(60) = -0.03(60)^2 + 60 + 48 = -108 + 60 + 48 = 0 \checkmark$$

Homework

Pg 52 #27-44